zad. 4 lista 2

$f(x) = 0.5x - 1 \qquad 2 < x \leq 4$

- $EX = \int_{-\infty}^{+\infty} x f(x)\,dx$

$$EX = \int_2^4 x(0.5x-1)\,dx = \int_2^4 0.5x^2 - x\,dx = \left[0.5\frac{x^3}{3} - \frac{x^2}{2}\right]_2^4 =$$

$$= \left[x^2\left(\frac{x}{6} - \frac{1}{2}\right)\right]_2^4 = \left[16\left(\frac{4}{6} - \frac{1}{2}\right) - 4\left(\frac{2}{6} - \frac{1}{2}\right)\right] = \frac{16}{6} + \frac{4}{6} = \frac{10}{3}$$

- $D^2X = \int_{-\infty}^{\infty} (x - EX)^2 f(x)\,dx$

$$D^2X = \int_2^4 (x - \frac{10}{3})^2 (0.5x - 1)\,dx =$$

$$= \int_2^4 (x^2 - \frac{20}{3}x + \frac{100}{9})(0.5x - 1)\,dx =$$

$$= \int_2^4 \frac{1}{2}x^3 - x^2 - \frac{20}{3}\cdot\frac{1}{2}x^2 + \frac{20}{3}x + \frac{100}{9}\cdot\frac{1}{2}x - \frac{100}{9}\,dx =$$

$$= \int_2^4 \frac{x^3}{2} - (1 + \frac{10}{3})x^2 + (\frac{60}{9} + \frac{50}{9})x - \frac{100}{9}\,dx =$$

$$= \int_2^4 \frac{x^3}{2} - \frac{13}{3}x^2 + \frac{110}{9}x - \frac{100}{9}\,dx =$$

$$= \left[\frac{x^4}{8} - \frac{13}{3}\frac{x^3}{3} + \frac{110}{9}\cdot\frac{x^2}{2} - \frac{100}{9}x\right]_2^4 =$$

$$= \frac{256}{8} - \frac{13}{9}\cdot 64 + \frac{55}{9}\cdot 16 - \frac{100}{9}\cdot 4 - \frac{16}{8} + \frac{13}{9}\cdot 8 - \frac{55}{9}\cdot 4 + \frac{100}{9}\cdot 2 = \frac{2}{9}$$

- $D^2X = EX^2 - (EX)^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - (EX)^2$

$$EX^2 = \int_2^4 x^2(0.5x-1)\,dx = \int_2^4 \frac{x^3}{2} - x^2\,dx = \left[\frac{x^4}{8} - \frac{x^3}{3}\right]_2^4 =$$

$$= \frac{256}{8} - \frac{64}{3} - \frac{16}{8} + \frac{8}{3} = \frac{34}{3}$$

$$(EX)^2 = \left(\frac{10}{3}\right)^2 = \frac{100}{9}$$

$$D^2X = EX^2 - (EX)^2 = \frac{34}{3} - \frac{100}{9} = \frac{102}{9} - \frac{100}{9} = \frac{2}{9}$$